ESD-TDR-65-74

Technical Note 1965-9

ESRL

The Two-Frequency Measurement of Line-of-Sight Spread Channels

M. L. Burrows

17 May 1965

Prepared under Electronic Systems Division Contract AF 19(628)-500 by

Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Lexington, Massachusetts

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the U.S. Air Force under Contract AF 19(628)-500.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

THE TWO-FREQUENCY MEASUREMENT OF LINE-OF-SIGHT SPREAD CHANNELS

M. L. BURROWS

Group 61

TECHNICAL NOTE 1965-9

17 MAY 1965

LEXINGTON MASSACHUSETTS

ABSTRACT

An analysis is carried out of the effects of various non-ideal circumstances on the two-tone method of measuring the two-frequency correlation function $R(\Omega, \tau)$ for microwave line-of-sight communication channels. These circumstances are the presence of both short-term and long-term instabilities of the signal sources, the presence of additive noise, and the use of discontinuous tones such as those available from a spinning satellite.

It is shown that oscillator instability effects are removable by assigning the short-term carrier oscillator instabilities to the channel, using high quality modulating oscillators and including phase-lock tracking loops, that the effect of additive noise can be made negligible provided the signal-to-noise power ratio of the received tones is greater than about 6 db, and that the use of discontinuous slowly-switched tones is distinctly disadvantageous.

A method of processing the received tones is suggested which automatically removes the large constant delay due to range, leaving only the frequency dependent part of the delay.

Accepted for the Air Force
Stanley J. Wisniewski
Lt Colonel, USAF
Chief, Lincoln Laboratory Office

The Two-Frequency Measurement of Line-Of-Sight Spread Channels

I. THE TWO-FREQUENCY METHOD

In order to transmit at a given information rate with less than a given error rate through a time-varying dispersive channel it is necessary, in general, that the signal element be carefully chosen and various diversity techniques employed. This aspect of the design of a communication link is made easier when a general channel characterization is known which is independent of the particular signal. One such characterization is the two-frequency correlation function $R(\Omega, \tau)$ defined as

$$R(\Omega, \tau) = E\{H^*(f - \frac{\Omega}{2}, t - \frac{\tau}{2}) H(f + \frac{\Omega}{2}, t + \frac{\tau}{2})\} \quad , \tag{1}$$

where H(f, t) is the time varying transfer function, E denotes ensemble average and the star denotes complex conjugate.

In general, $R(\Omega, \tau)$ is also a function of f and t, but for line-of-sight atmospheric communication paths in the microwave region it is certainly convenient and yet not too restrictive to assume that $R(\Omega, \tau)$ is very weakly dependent upon f and t compared with its dependence on Ω and τ. This means that when $R(\Omega, \tau)$ is measured, it is necessary only to specify the band of the microwave spectrum in which f lies and the observable weather conditions that prevailed at the time of the measurement.

A direct method of measuring $R(\Omega, \tau)$ is to transmit two tones,

$x_n(t) = \exp\{j2\pi f_n t\}$, $(n = 1, 2)$ over the path. These are separated from one another at the receiver by filtering and then time-correlated with frequency offset $f_2 - f_1$. The result is

$$< y_1^*(t - \frac{\tau}{2})\, y_2(t + \frac{\tau}{2})\, e^{-j2\pi(f_2 - f_1)t} > = < H^*(f_1, t - \frac{\tau}{2})\, H(f_2, t + \frac{\tau}{2}) > e^{j\pi(f_2 + f_1)\tau}$$

$$= R(f_2 - f_1, \tau)\, e^{j\pi(f_2 + f_1)\tau} \quad , \qquad (2)$$

where the further assumption of ergodicity has been made to enable the time average, denoted by $<>$, to be replaced by expected value. This result derives directly from the definition of $H(f, t)$, which is that $H(f, t) \exp\{j2\pi ft\}$ is the received signal when a signal $\exp\{j2\pi ft\}$ is transmitted.

First, we observe that the transmitted signal is not a pure tone and so we write

$$x_n(t) = a_n(t)\, e^{j2\pi f_n t} \qquad (3)$$

where f_n is the nominal frequency and $a_n(t)$ describes the net effect of all possible modulating disturbances e.g., frequency offset and drift, random frequency jitter, switching functions, if any, and Doppler effects.

The received signal $y_n(t)$ is given by

$$y_n(t) = \int_{-\infty}^{\infty}\int H(f, t)\, x_n(t')\, e^{j2\pi f(t - t')} dt' df \quad , \tag{4}$$

and after mixing with $b_n(t) \exp\{j2\pi f_n t\}$ to reduce the nominal frequency from f_n to zero, the resulting signal $z_n(t)$ is given by

$$z_n(t) = y_n(t)\, b_n^*(t) \quad , \tag{5}$$

where $b_n(t)$ describes the net modulation of the local oscillator. (It is probably more convenient in practice to reduce the frequency to some common frequency different from zero, but it is certainly possible, by quadrature processing, to choose a common frequency of zero and it is more convenient analytically.)

According to Eq. (2), $R(f_m - f_n, \tau)$ is measured by determining the time average of $z_n^*(t - \tau/2)\, z_m(t + \tau/2)$. An expression for this average may be obtained by combining Eqs. (3), (4) and (5) and then carrying out a series of transformations. A simpler way is to observe that since $x_n(t)$ is a narrow band signal and since the channel is a microwave line-of-sight communication path, then over the band occupied by $x_n(t)$, $H(f, t)$ can be written as $H(f_n, t) \exp\{-j2\pi r_n (f - f_n)/c\}$, where r_n is the effective range at frequency f_n and c is the speed of light. Then, from Eq. (3) and Eq. (4), we obtain

$$y_n(t) = H(f_n, t)\, a_n(t - \frac{r_n}{c})\, e^{j2\pi f_n t} \quad , \tag{6}$$

which, together with Eq. (5), yields the equation

$$< z_n^*(t - \frac{\tau}{2}) \, z_m(t + \frac{\tau}{2}) >$$

$$= < H^*(f_n, t - \frac{\tau}{2}) \, H(f_m, t + \frac{\tau}{2}) \, a_n^*(t - \frac{r_n}{c} - \frac{\tau}{2}) \, a_m(t - \frac{r_m}{c} + \frac{\tau}{2}) \, b_n(t - \frac{\tau}{2}) \, b_m^*(t + \frac{\tau}{2}) > \tag{7}$$

It is quite reasonable to assume that the statistics of the terminals and of the channel are independent, so that the time average of the product can be written as the product of the time averages. In addition, from the assumption of ergodicity for the channel, $< H^*(f_n, t - \frac{\tau}{2}) \, H(f_m, t + \frac{\tau}{2}) >$ can be written as $R(f_m - f_n, \tau)$. Thus Eq. (7) reduces to

$$< z_n^*(t - \frac{\tau}{2}) \, z_m(t + \frac{\tau}{2}) > = R(f_m - f_n, \tau) < F_{nm}(t, \tau) > \tag{8}$$

where

$$F_{nm}(t, \tau) = a_n^*(t - \frac{r_n}{c} - \frac{\tau}{2}) \, a_m(t - \frac{r_m}{c} + \frac{\tau}{2}) \, b_n(t - \frac{\tau}{2}) \, b_m^*(t + \frac{\tau}{2}) \quad .$$

Now each "modulating function," $a_n(t)$ or $b_n(t)$, can be factored as

$$a_n(t) = \alpha_n(t) \, A_n(t) \quad ,$$

$$b_n(t) = \beta_n(t) \, B_n(t) \quad ,$$

where α_n, β_n are the "random" parts of a_n, b_n and A_n, B_n the "deterministic" parts. The intention here is to separate the short-term oscillator instabilities

and other jitter effects from frequency offset, long-term oscillator drift and switching functions. The ultimate specification of the factoring process is defined by the manner in which the measuring equipment distinguishes the factors.

We assume that $\alpha_n(t)$ and $\beta_b(t)$ are sample functions from independent stationary ergodic stochastic processes. Then Eqs. (8) and (9) can be written as

$$< z_n^*(t - \frac{\tau}{2})\, z_m(t + \frac{\tau}{2}) > = R(f_m - f_n, \tau)\, R_{nm}^{(\alpha)}(\tau)\, R_{nm}^{*(\beta)}(\tau) < G(t, \tau) > \qquad (10)$$

where $R_{nm}^{(\alpha)}(\tau)$ is the complex cross-correlation function between $\gamma_n(t)$ and $\gamma_m(t)$, $(\gamma = \alpha, \beta)$, and $G(t, \tau)$ is given by

$$G(t, \tau) = A_n^*(t - \frac{r_n}{c} - \frac{\tau}{2})\, A_m(t - \frac{r_m}{c} + \frac{\tau}{2})\, B_n(t - \frac{\tau}{2})\, B_m^*(t + \frac{\tau}{2}) \qquad (11)$$

Thus the success of the two-frequency approach to the measurement of the channel characteristics depends upon the extent to which the short-term effects, represented by the factors $R_{nm}^{(\alpha)}(\tau)$ and $R_{nm}^{*(\beta)}(\tau)$ in (10), contaminate the result, and also upon the errors made at the receiver in estimating the deterministic features of the transmitted signals, represented by the factor $< G(t, \tau) >$.

II. SHORT-TERM INSTABILITIES

One possible method of generating the tones is to use a single RF or

carrier oscillator modulated with various multiples of a single reference oscillator. Thus, if the instantaneous phases of the two oscillators be represented by $\phi_c(t)$ and $\phi_o(t)$, then $\alpha_n(t)$ is given to within an arbitrary constant factor by

$$\alpha_n(t) = \exp\{j\phi_c(t) + jn\phi_o(t)\} \tag{12}$$

since the output will be stabilized in amplitude.

An indication of the effect of random variations in $\phi_c(t)$ and $\phi_o(t)$ is obtained by assuming that the statistics of the phases are independent and Gaussian, then

$$\begin{aligned} R^{(\alpha)}_{nm}(\tau) &= E\{\alpha_n^*(t - \tfrac{\tau}{2})\, \alpha_m(t + \tfrac{\tau}{2})\} \\ &= R_c(\tau)\{R_o(\tau)\}^{mn}\, e^{-\frac{1}{2}(m-n)^2\sigma_o^2} \end{aligned} \tag{13}$$

where $R_c(\tau)$ and $R_o(\tau)$ are the auto-correlation functions of the output signals from the carrier and reference oscillators respectively, σ_o^2 is the variance of the reference oscillator phase, and the mean phases have been absorbed in the arbitrary constant factor. (The notation $\{R_o(t)\}^{mn}$ has the conventional interpretation, viz., $R_o(\tau)$ raised to the power m times n.)

A similar expression is obtained for $R^{(\beta)}_{nm}(\tau)$ when the same assumptions are made for $\beta_n(t)$.

Equation (13) demonstrates the two ways in which the effects of oscillator instability is manifested. The first is the reduction in the range of τ for

which $R(f_m - f_n, \tau)$ can be measured (high quality microwave sources have spectral widths typically of the order of 1 Hz, so that $R_c(\tau)$ will be essentially zero for $|\tau|$ greater than a few tenths of a second). The second is the reduction in the range of $f_m - f_n$ for which $R(f_m - f_n, \tau)$ can be measured, although this does not appear to be serious for values of $f_m - f_n$ of importance in communication links. (For example, if a value of $f_m - f_n$ of 100 MHz is attained by multiplying up the signal from a 5 MHz reference oscillator, then the factor $\exp\{-(m-n)^2\sigma_o^2/2\}$ is not less than 0.9 provided σ_o is not greater than about 25 milli-radians, which is well within the capability of high quality HF sources.)

III. DETERMINISTIC EFFECTS

There are two types of deterministic effects of concern here. One involves the errors made at the receiver in estimating the frequency and the rate of drift of frequency of the tones, and the other is concerned with the limitations imposed on the measurement by the fact that each tone may be present for only a part of the time.

If the tone frequency and drift rate are f_n and α_n respectively, and the estimates of the receiver of these quantities are f_n' and α_n' respectively then clearly unless $f_m' - f_n' = f_m - f_n$ and $\alpha_m' - \alpha_n' = \alpha_m - \alpha_n$ exactly, the time average of $G(t, \tau)$ [see Eq. (11)] is strictly zero. In practice, of course, a finite time of averaging is used, so that some error can be tolerated. The obvious way to ensure that the estimates are accurate is to employ phase-

lock tracking, with large time constant, at the receiver to derive a "detrended" version of each received tone.

If each "tone" is a slowly switched pulse train rather than being continuous, and the inter-pulse period is greater than the coherence time of the channel fluctuations, then phase-lock tracking is impossible. In this case a possibility is to process the signals by carrying out the finite time average using a range of values of $f_m' - f_n'$ and $\alpha_m' - \alpha_n'$ and then taking the maximum of the resulting surface as the required estimate. This two dimensional processing must then be repeated for each measuring point in the $f_m - f_n$, τ plane which entails a large amount of signal processing. However, since this searching operation is effectively just another way of phase-lock tracking, then if the one is impossible it is unlikely that the other will succeed.

Thus, for a successful measurement, it appears that the switching period of the tone samples or pulses must be less than the coherence time of the channel fluctuations. At the even higher switching rate of more than twice the bandwidth of the channel fluctuations, a single Fourier component of the received signal can be selected by filtering and then processed as a separate continuous tone.

There are three other penalties imposed by the use of slowly switched tones, even if the rate is high enough for phase-lock tracking. These are (a) that the tone samples must be tracked with a time gate, (in order that the maximum signal-to-noise ratio be obtained), (b) that the resulting measure-

ment of $R(f_m - f_n, \tau)$ is possible only at intervals of τ equal to the switching rate, and (c), that the averaging time must be increased if the same variance in the measurement is to be maintained. Cases (a) and (b) are self evident. Case (c) is dealt with in the next section.

IV. EFFECT OF NOISE AND FINITE AVERAGING TIME

In this section we assume that the deterministic variations are not present except for simple on - off periodic switching functions, that the short-term equipment instabilities are negligible and that now additive white noise is present. Thus, when a switched tone of amplitude a_o is transmitted, the received signal is, after gating and reduction to zero frequency,

$$u_n(t) = s_n(t) \{a_o H(f_n, t) + w_n(t)\} \tag{14}$$

where $s_n(t)$ is the slow-rate periodic gating function (which is synchronized with and of shorter duration than the switching function in the transmitter, so that the latter does not appear) and $w_n(t) \exp\{j2\pi f_n t\}$ is the additive white noise associated with the n'th tone.

The measurement depends upon the finite time average

$$U_{nm}(\tau) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_n^*(t - \tfrac{\tau}{2})\, u_m(t + \tfrac{\tau}{2})\, dt \quad . \tag{15}$$

If we focus attention on those values of τ (say τ_i) for which $s_n(t - \frac{\tau}{2})$ and

$s_m(t + \frac{\tau}{2})$ are coincident, then

$$E\{k^{-1} U_{nm}(\tau_i)\} = a_o^2 R(f_m - f_n, \tau_i) + R^{(w)}_{nm}(\tau_i) \quad , \tag{16}$$

where

$$k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s_n(t - \frac{\tau_i}{2})\, s_m(t + \frac{\tau_i}{2})\, dt$$

and $$R^{(w)}_{nm}(\tau) = E\{w_n^*(t - \frac{\tau}{2})\, w_m(t + \frac{\tau}{2})\}$$

$$= 0, \text{ for } n \neq m \quad .$$

For our estimate of $R(f_m - f_n, \tau_i)$, therefore, we take $a_o^{-2}\{k^{-1} U_{nm}(\tau_i) - R^{(w)}_{nm}(\tau_i)\}$, where the noise contribution $R^{(w)}_{nm}(\tau_i)$ is different from zero only when $n = m$. It can either be estimated directly by measuring $U_{nn}(\tau_i)$ in the absence of a transmitted signal, or be excluded by basing the resulting plot of $R(f_m - f_n, \tau)$ only on those measurements of $U_{nm}(\tau)$ for which $n \neq m$.

The variance $\sigma^2(\tau_i)$ of the estimate is computed in the following manner

$$\sigma^2(\tau_i) = E\left\{ \left| \frac{U_{nm}(\tau_i)}{a_o^2 k} - \frac{R_{nm}(\tau_i)}{a_o^2} - R(f_m - f_n, \tau_i) \right|^2 \right\} \quad . \tag{17}$$

By assuming that the channel statistics and the noise statistics are independent and that both are multivariate Gaussian of zero mean, Eq. (17) can be written

$$\sigma^2(\tau_i) = \frac{1}{a_o^4 k^2 T^2} \int\int_{-\frac{T}{2}}^{\frac{T}{2}} \Big[a_o^4 |R(0, t'-t)|^2 + a_o^2 R(0, t'-t)\, R_{mm}^{*(w)}(t'-t)$$

$$+ a_o^2 R^*(0, t'-t)\, R_{nn}^{(w)}(t'-t) + R_{nn}^{(w)}(t'-t)\, R_{mm}^{*(w)}(t'-t) \Big]$$

$$\times\, s_n(t' - \frac{\tau_i}{2})\, s_m(t' + \frac{\tau_i}{2})\, s_n(t - \frac{\tau_i}{2})\, s_m(t + \frac{\tau_i}{2})\, dt\, dt' \qquad (18)$$

Now the four-fold product of the gating functions in the integrand is, for the special values τ_i, a doubly periodic array of square columns standing on the t, t' plane. If the density of these columns is high, that is, if the period of the gating functions is short compared with the width in τ of $R(0, \tau)$ and $R_{nn}^{(w)}(\tau)$, then the effect of the gating on the integral is merely to modify it by the constant factor k^2. The resulting integral may then be transformed to give

$$\sigma^2(\tau_i) = \frac{1}{a_o^4 T} \int_{-T}^{T} \left| 1 - \frac{\tau}{T} \right| \Big\{ a_o^4 |R(0, \tau)|^2 + a_o^2 R(0, \tau)\, R_{mm}^{*(w)}(\tau)$$

$$+ a_o^2 R^*(0, \tau)\, R_{nn}(\tau) + R_{nn}^{(w)}(\tau)\, R_{mm}^{*(w)}(\tau) \Big\} dt \quad .$$

Therefore if T is large compared with the spread in τ of $R(0, \tau)$ and $R_{nn}^{(w)}(\tau)$, then $\sigma^2(\tau_i)$ is equal to the sum of four terms of the type $\int R_1(\tau) R_2^*(\tau)\, d\tau$, which is equal to $\int S_1(f) S_2(f)\, df$, where $S_1(f)$ and $S_2(f)$ are the spectral densities corresponding to the correlation functions $R_1(\tau)\, R_2(\tau)$.

The spectral density of the noise can be assumed to be N_o over a band B, the bandwidth of the receiver for each tone, and zero elsewhere. The spectral density of the channel variations may, for illustrative purposes, be approximated by the Gaussian shape $A \exp\{-2f^2/B_c\}$, so that B_c is roughly the 2 db bandwidth of the channel fluctuations. Then since B must be significantly greater than B_c if the tones are to be processed without distortion, we obtain approximately

$$\sigma^2(\tau_i) \approx \frac{1}{T}\left[\frac{R^2(0, 0)}{\sqrt{\pi}\, B_c} + \frac{2N_o R(0, 0)}{a_o^2} + \frac{N_o^2 B}{a_o^4}\right] .$$

But $a_o^2 R(0, 0)$ is the received power S, say, so that the standard deviation normalized with respect to $R(0, 0)$, the maximum of $R(\Omega, \tau)$, can be written as

$$\frac{\sigma(\tau_i)}{|R(0, 0)|} \approx \frac{1}{\sqrt{\pi^{1/2} T B_c}} \left\{1 + 2\sqrt{\pi}\left(\frac{N}{S}\right)\frac{B_c}{B} + \sqrt{\pi}\left(\frac{N}{S}\right)^2 \frac{B_c}{B}\right\}^{1/2} , \tag{19}$$

where $S/N = S/N_o B$, the signal-to-noise power ratio for each received tone. Therefore, since $B_c < B$, when the signal-to-noise ratio is significantly greater than unity, the factor in curly brackets is close to unity and the normalized standard deviation of the measured value of $R(f_m - f_n, \tau_i)$ is roughly $(\pi^{1/2} T B_c)^{-1/2}$. Thus, there is little to be gained by striving to increase the signal-to-noise ratio above, say, 6 db.

Equation (19) has been obtained from Eq. (18) by using the assumption that the period of the gating functions is short compared with the width in τ of $R(0, \tau)$ and $R_{nn}^{(w)}(\tau)$. If we now go to the other extreme and assume that the period is long, then only the gating columns lying closest to the diagonal will be effective. On the assumption that the width of each gate is small and that $s_n(t) = s_m(t)$, we obtain in place of Eq. (19)

$$\frac{\sigma(\tau_i)}{|R(0, 0)|} \approx \frac{1}{\sqrt{T f_g}} \left(1 + \frac{N}{S}\right) \tag{19a}$$

where f_g is the gate repetition frequency.

As before, the optimum value of S/N is of the order of 6 db but now the standard deviation is inversely proportional to the square root of the number $(T f_g)$ of samples, which is to be expected since the assumption that the samples are independent is implicit in the assumption that the switching rate is slow. This assumption also means that $f_g < B_c$, and therefore, from Eqs. (19) and (19a), that the standard deviation is larger for the slowly switched

case.

The probable magnitude of B_c, the bandwidth of the channel fluctuations, is of the order of a few Herz. Therefore, if the normalized standard deviation is to be reduced to 0.1, integration times of the order of a few tens of seconds must be used. This estimate also illustrates the necessity of accurate frequency matching between the locally generated tones and the received tones, for the period of the frequency match error must be long compared with the integration time.

The average total power output of the transmitter must be divided between the tones. It may be advantageous to switch through the tones sequentially rather than radiate all tones continuously. In that case the switching period for each tone must be less than the Nyquist interval of the channel fluctuations. This ensures that the frequency spread imposed by the channel on each Fourier component of the switched tones does not overlap the spread imposed on the neighboring tones. Then the spread central Fourier component can be selected without distortion by filtering.

The question arises of whether an improved signal-to-noise ratio can be attained by making the radiated tone samples very short and of high intensity. It is easy to show that if P is the average power of each tone and the "tone" is radiated for a fraction $1/m$ of the time, then after the central Fourier component has been selected by filtering, the resulting continuous signal has

an average power given by P/m. (The peak power is mP but the power in the central component is $1/m^2$ of this.) Therefore, if N_o is the additive white noise power per unit bandwidth, the signal-to-noise power ratio is $P/m\, N_o B$ if no pulse gating is used at the receiver and is $P/N_o B$ if gating is used. Here B is the bandwidth of the receiver for each tone.

The answer to the question, therefore, is that no improvement is possible by using very short high intensity pulses, since the best obtainable signal-to-noise power ratio $P/N_o B$ is independent of m.

V. PHASE CONTROL

Measurement of the two frequency correlation function involves the measurement of both amplitude and phase. Thus, it is necessary to keep close control of the phase relationships between the tones. Clearly, if each tone is separately tracked by a phase-lock loop, the differences in phase between the tones are eliminated and no meaningful phase measurement can be made.

However, if a multiple tone transmission is used in which the phase interrelationships between the tones is fixed, it seems that it is possible to use phase-lock tracking and still retain the required phase information. For example, the transmitted tones may be generated by using a single carrier oscillator mixed with various integral multiples of the output from a single reference oscillator. At the receiver, the necessary local oscillator tones would be generated using an identical mixing procedure to that used for gen-

erating the tones in the transmitter. The two oscillators in the receiver, however, would be voltage controlled by phase-lock tracking just two of the transmitted tones with the two corresponding locally generated tones. Then if the channel were strictly linear phase, the remaining tones would also be effectively phase locked, but any deviation from a linear phase law would manifest itself as phase errors between the remaining unlocked tones and their corresponding locally generated tones.

This method has the interesting property that range effects are automatically taken into account so that there is no need to estimate the expected (large) number of phase rotations between one tone and the next in order to obtain the particular phase interval in which to place the measured phase. In practical terms the method has the effect of removing the constant time delay due to range, which is of no interest in any case, and leaving only the frequency dependent part of the delay.

The method is examined in detail in the Appendix where it is shown that the two tones chosen for tracking must be the two with the smallest frequency difference between them. If this is not done, ambiguity can arise in the definition of the phase of some of the remaining tones. It is also shown in the Appendix that the full benefits of the method cannot be attained when just two tones are transmitted during each time averaging interval, even if the two tones are derived from a single oscillator.

The fact that this method requires the simultaneous transmission of at

least three of the full set of tones required for the complete measurement is not a disadvantage of the method because the variability of weather conditions makes it necessary to carry out the complete measurement in as short a time as possible. Thus multiple-tone transmission is required in any case.

It is necessary, of course, to compensate for any difference in phase rotation occurring between the various tone channels.

VI. CONCLUSION

In view of the assumption of stationarity, which is basic to the characterization of the channel by the two-frequency correlation function, it is important that all the measurements necessary to plot the function be made simultaneously and in as short a time as possible. The two-frequency method of measurement is well suited to these requirements because a large number of tones can be transmitted simultaneously and compared in pairs to cover the whole range of frequency differences of interest. Also, since the receiver bandwidth can be made as small as the net uncertainty in frequency of the received tones, the method can discriminate well against additive noise and therefore make do with a short averaging time to achieve the desired variance in the measurements.

The facts that the transmitter may be moving with respect to the receiver and that the frequencies of the transmitted tones are not known exactly at the receiver make it necessary to carry out some frequency estimating procedure at the receiver. The only practicable way of dealing with this problem appears

to be phase-lock tracking.

If phase-lock tracking is used on every received tone phase information is lost. However, it has been shown by using a suitable method of tone generation and a selective use of phase-lock tracking all useful phase information can be measured directly since the pure-delay linear phase is cancelled out.

An objection to the choice of the two-frequency method of measurement might be that oscillator instabilities (and especially those of the carrier oscillator) preclude any accurate measurement for values of τ other than $\tau = 0$. However, it is not possible to use the channel without a carrier oscillator and so it is quite in order to define the "channel" to be the combination of modulation, transmission and demodulation operations taken together. Then, since the time correlation of the channel is as least as bad as that of the carrier oscillator, it should be easy to attain sufficient short-term stability in the other lower frequency oscillators involved in the measurement.

Finally, the further problems associated with the use of slowly switched tones have been examined in some detail. This is because it is necessary to have either a full-stabilized vehicle or an elaborate antenna structure for a satellite source to appear non-fluctuating at the ground. If the measurement can be made when the source is single narrow-beam antenna mounted on a spin-stabilized satellite the experiment is much easier to implement.

The results show that the effects of using such "slowly-switched" signals are:

(1) Phase-lock tracking is no longer possible, which means that extensive data processing is necessary at each measured point to search for the appropriate frequency drift and offset,

(2) Measurements can be made only at values of τ equal to integral multiples of the switching period,

(3) Time-gate tracking is necessary in order to maintain the maximum signal-to-noise ratio,

(4) The averaging time necessary to achieve a given variance in the measurements is increased.

Thus the use of continuous tones (or tones sampled at intervals no larger than the Nyquist interval) is much to be preferred.

APPENDIX

This appendix demonstrates the possibility of using phase-lock tracking in such a way that essential phase information is retained and the constant range delay is automatically cancelled.

We consider a set of transmitted tones generated by mixing multiples of a reference oscillator signal with a carrier oscillator. Then the phase of each tone may be written as

$$\mathrm{Arg}\{x_n(t)\} = \varphi_c(t) + n\varphi_o(t) \tag{A-1}$$

where, since we are concerned here solely with the slowly varying or "deterministic" variation, $\varphi_c(t)$ and $\varphi_o(t)$ are the smoothed instantaneous phases of the carrier oscillator and the reference oscillator, respectively.

By the time the tone reaches the receiver it has received a phase shift which is approximately proportional to the frequency, and so the phase of the received tone can be expressed as

$$\mathrm{Arg}\{y_o(t)\} = \varphi_c(t) + n\varphi_o(t) - [\varphi_c'(t) + n\varphi_o'(t)]\rho_n, \tag{A-2}$$

where the primes denote differentiation with respect to t (so that $\varphi_c'(t) + n\varphi_o'(t)$ is 2π times the smoothed frequency) and ρ_n is a factor weakly dependent upon both the frequency and the time. Thus ρ_n may be regarded as the effective frequency - dependent instantaneous range divided by the speed of light. The suffix n denotes that ρ_n is evaluated at the frequency $\varphi_c'(t) + n\varphi_o'(t)$.

At the receiver a similar set of tones is generated to provide the local

oscillator signals. The phase of these tones may be written as

$$\mathrm{Arg}\{b_n(t)\} = \psi_c(t) + n\psi_o(t) \quad , \tag{A-3}$$

where $\psi_c(t)$ and $\psi_o(t)$ are the phases of the receiver oscillators corresponding to the carrier and reference oscillators in the transmitter, respectively.

The receiver oscillators are voltage controlled by means of phase-lock loops which lock the phases of two of the locally generated tones with the smoothed phases of their corresponding received tones. We may choose, without any loss of generality, to lock $b_o(t)$ and $b_g(t)$ with $y_o(t)$ and $y_g(t)$. That is, we adjust $\psi_c(t)$ and $\psi_o(t)$ to satisfy the equations

$$\mathrm{Arg}\{b_o(t)\} = \mathrm{Arg}\{y_o(t)\} \quad ,$$

$$\mathrm{Arg}\{b_g(t)\} = \mathrm{Arg}\{y_g(t)\} \quad .$$

Written in full these equations are, from (A-2) and (A-3),

$$\psi_c(t) = \varphi_c(t) - \varphi_c'(t)\,\rho_o$$

$$\psi_c(t) + g\psi_o(t) = \varphi_c(t) - \varphi_c'(t)\,\rho_g + g\{\varphi_o(t) - \varphi_o'(t)\,\rho_g\} + k2\pi \quad .$$

The $k2\pi$ term is necessary, where k is any integer, because any integral multiple of $2\pi/g$ can be added to $\psi_o(t)$ without altering the conditions for phase lock.

By solving these equations for $\psi_c(t)$ and $\psi_o(t)$ we find that the phase of $b_n(t)$ is given by

$$\operatorname{Arg}\{b_n(t)\} = \varphi_c(t) + n\varphi_o(t) + \frac{nk}{g} 2\pi - \varphi_c{}'(t) \frac{g\rho_o + n\rho_g - n\rho_o}{g} - n\varphi_o{}'(t)\,\rho_g \quad . \tag{A-4}$$

Therefore when the received tones are reduced to a nominal frequency of zero by mixing with their corresponding locally generated tones given by Eq. (A-4), we find for the phase of the resulting signals

$$\begin{aligned}\operatorname{Arg}\{z_n(t)\} &= \operatorname{Arg}\{y_n(t)\} - \operatorname{Arg}\{b_n(t)\} \\ &= -\{\varphi_c{}'(t) + n\varphi_o{}'(t)\}\,\rho_n + \varphi_c{}'(t)\frac{g\rho_o + n\rho_g - n\rho_o}{g} + n\varphi_o{}'(t)\,\rho_g \\ &\quad - \frac{nk}{g} 2\pi \quad .\end{aligned} \tag{A-5}$$

From Eq. (A-5) we see that $\operatorname{Arg}\{z_o(t)\} = 0$ and $\operatorname{Arg}\{z_g(t)\} = -k2\pi$, so that the measured phase will be zero for $n = 0$ and $n = g$, as it should be since these values of n are the reference points of the phase control. Further, if the medium is non-dispersive then ρ_n will be independent of n and in this case $\operatorname{Arg}\{z_n(t)\}$ will be equal to $-nk2\pi/g$. Thus if g is chosen to be unity, $\operatorname{Arg}\{z_o(t)\}$ will be measured as zero in the non-dispersive case for all n. Therefore, we choose $x_o(t)$ and $x_1(t)$ as our reference tones, and Eq. (A-5) reduces to

$$\operatorname{Arg}\{z_n(t)\} = -\{\varphi_c{}'(t) + n\varphi_o{}'(t)\}\,\rho_n + \varphi_c{}'(t)\,(\rho_o + n\rho_1 - n\rho_o) + n\varphi_o{}'(t)\,\rho_1 \quad . \tag{A-6}$$

That this expression does in fact equal the phase of $z_n(t)$ relative to the linear phase law defined by the reference points is easily tested. If we represent the phase difference over the channel for the n'th tone as θ_n then θ_n is given by $\theta_n = -\{\varphi_c'(t) + n\varphi_o'(t)\}\,\rho_n$. The straight line phase $\tilde{\theta}_n$ based on θ_o and θ_1 as reference points is given by $\tilde{\theta}_n = \theta_o + n(\theta_1 - \theta_o)$, so that the relative phase difference $\Delta\theta$ between the actual phase θ_n and the linear phase $\tilde{\theta}_n$ is given by $\Delta\theta = \theta_n - \theta_o - n(\theta_1 - \theta_o)$. When the full expressions for the θ_n are substituted in this equation it becomes identical to Eq. (A-6).

It is easy to show that if a pulse of bandwidth B is transmitted through a dispersive channel having a quadrature component in the phase law, then the onset of severe distortion occurs when the deviation of the phase law from a linear law over this bandwidth is of the order of one radian. Since we are interested in the measurement of communication channels, therefore, no ambiguity in the measurement of the phase should arise.

If the channel is strictly stationary it is possible in principle to measure the two-frequency correlation function by transmitting a pair of tones which are changed in frequency after each averaging interval. By this means a better signal-to-noise ratio is obtained since the total power is divided between only two tones.

Clearly, a phase-lock system of the type described above will not work in this case because once both tones have been used for locking, all deterministic phase information is destroyed. However, if only one oscillator is

used to generate both tones there is just one oscillator to be locked at the receiver and the possibility arises of achieving the same performance as in the previous case.

The phase of the received tones is $n\{\varphi_o(t) - \varphi_o'(t)\, \rho_n\}$, the phase of the locally generated tones is $n\psi_o(t)$ and so the equation to be satisfied for phase locking on one of the tones is

$$g\psi_o(t) = g\{\varphi_o(t)) - \varphi_o'(t)\, \rho_g\} + k2\pi \quad .$$

This equation gives $\psi_o(t)$ directly and so the result of mixing the other received tone with its corresponding locally generated tone is the phase $- n\varphi_o'(t)\, (\rho_n - \rho_g) - nk2\pi/g$. Since it is not possible in this case to choose g equal to unity (in fact, g must be a large integer) there remains the irreducible phase uncertainty $nk2\pi/g$. Thus, we conclude that when the tones are transmitted two at a time the phase lock technique cannot be used to full advantage.

DISTRIBUTION LIST

Division 6

Dinneen, G. P.
Morrow, W. E.

Group 61

Ricardi, L. J. (15)
Assaly, R. N.
Burrows, M. L. (10)
Crane, R. N.
Devane, M. E.
LaPage, B. F.
Lindberg, C. A.
Rankin, J. B.
Rosenthal, M. L.
Sotiropoulos, A.
Niro, L.
Peck, R. J.

Group 62

Rosen, P. (2)

Group 63

Sherman, H. (2)

Group 64

Green, P. E.

Group 65

Wood, R. V.

Group 66

Reiffen, B.

Division 4

Weiss, H. G.

Group 44

Allen, J. L.

Group 45

Ward, W. W.

Group 46

Jones, C. W.
Keeping, K. J.

Division 3

Chisholm, J. H.
Ruze, J.

Division 7

Hutzenlaub, J. F.

Group 71

Blaisdell, E. W.

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Lincoln Laboratory, M.I.T.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP None

3. REPORT TITLE

The Two-Frequency Measurement of Line-of-Sight Spread Channels

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Note

5. AUTHOR(S) *(Last name, first name, initial)*

Burrows, Michael L.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
17 May 1965	34	None

8a. CONTRACT OR GRANT NO. AF 19(628)-500	9a. ORIGINATOR'S REPORT NUMBER(S) TN-1965-9
b. PROJECT NO. c. None d.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* ESD-TDR-65-74

10. AVAILABILITY/LIMITATION NOTICES

None

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Air Force Systems Command, USAF

13. ABSTRACT

An analysis is carried out of the effects of various nonideal circumstances on the two-tone method of measuring the two-frequency correlation function $R(\Omega, \tau)$ for microwave line-of-sight communication channels. These circumstances are the presence of both short-term and long-term instabilities of the signal sources, the presence of additive noise, and the use of discontinuous tones such as those available from a spinning satellite.

It is shown that oscillator instability effects are removable by assigning the short-term carrier oscillator instabilities to the channel, using high quality modulating oscillators and including phase-lock tracking loops, that the effect of additive noise can be made negligible provided the signal-to-noise power ratio of the received tones is greater than about 6 db, and that the use of discontinuous slowly switched tones is distinctly disadvantageous.

A method of processing the received tones is suggested which automatically removes the large constant delay due to range, leaving only the frequency dependent part of the delay.

14. KEY WORDS

frequency measurement
communication channels